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MATHEMATICAL ASSOCIATION OF AMERICA--COMMITTEE ON THE UNDERGRADUATE PROGRAM IN MATHEMATICS--PANEL ON PREGRADUATE TRAINING

PRELIMINARY RECOMMENDATIONS FOR AN HONORS PROGRAM IN MATHEMATICS

MATHEMATICAL ASSOCIATION OF AMERICA

PRELIMINARY RECOMMENDATIONS

FOR

AN HONORS PROGRAM

IN

MATHEMATICS

COMMITTEE ON THE UNDERGRADUATE
PROGRAM IN MATHEMATICS

JUNE, 1962

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The Committee on the Undergraduate Program in Mathematics, supported by the National Science Foundation and the Ford Foundation, is a committee of the Mathematical Association of America, charged with making recommendations for the overall improvement of college and university mathematics curricula at all levels and in all educational areas. Moreover, the Committee devotes its energies to reasonable efforts in realizing these recommendations.

Membership of the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America

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To reflect one of its major concerns, the Committee has established a Panel on Pre-Graduate Training with additional mathematicians to aid with deliberations, and this Panel is comprised as follows:

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Johns Hopkins University

The Panel on Pre-Graduate Training began in 1960 by constructing an idealized program in terms of what might be suitable for honors students in mathematics, and the purpose of this document is to report the resultant recommendations of this Panel, which have also received the approval of the full Committee.

INTRODUCTION

This preliminary report outlines an honors program in mathematics for undergraduate students. Although few institutions will wish to follow the program herein set forth in every detail, the proposal will nevertheless serve the following purposes:

1. It indicates the level of mathematical maturity desirable for the beginning graduate student who expects to create, teach, or apply the central disciplines of mathematics: algebra, analysis, and geometry.
2. It indicates changes that are now feasible in some colleges and universities, and which can be achieved in the near future at others.
3. It helps to locate the areas in which new textual material and new courses are needed.
4. It serves as a basis for designing curricula suitable for majors whose interests lie largely in applications, as well as for those outside the honors group.

Although this report is concerned primarily with mathematical content, the Panel is aware that there are other essential aspects of an effective program. A mere collection of material, however rich and full it is mathematically, cannot guarantee understanding on the part of the student. We return to this important question at the end of the report.

In developing the program, we have taken for granted the need for closing the gap between undergraduate instruction and contemporary mathematics. The gap alluded to is not simply a matter of level or advancement. More than that, we are concerned with a gulf between the spirit and outlook of the undergraduate program and the current literature in mathematics. The trend that we urge, which is, of course, already observable at many colleges and universities, is facilitated by the new programs in secondary schools. Indeed, our proposal presupposes that the student has the background of the School Mathematics Study Group program in secondary school, or the equivalent. Thus, it is presumed that the student entering college with the intention of majoring in mathematics is prepared to begin a serious course in calculus — a body of theorems, not simply a collection of devices and procedures for solving a certain class of problems.

THE PROGRAM

I. Introductory Undergraduate Mathematics

Whereas this report as a whole pertains to the programs of students majoring in mathematics, the introductory undergraduate courses we propose are suitable for a much wider class of honors students, namely all those seeking a solid foundation of basic mathematics. We recommend that for such students the beginning college program be devoted to a study of calculus, analytic geometry, linear algebra, and differential equations, with the main emphasis on calculus as in the traditional program. This comprises five semesters at three hours per week. Since the topics are integrated, it is not possible to make a clear separation into semester units. However, the proportions are roughly three semesters of calculus and analytic geometry, one semester of linear algebra, and one semester of differential equations. Since it totals five semesters, the work might well extend into the junior year in many institutions. (For schools not on the semester plan, our proposal should be subjected to the obvious transformation.) The three semesters of calculus encompass the traditional year course in the calculus of one variable and also topics in "advanced calculus," so that the calculus of functions in n-dimensional space is available in the early college years.

There are variations possible in the arrangement of all this material, partly because there is some flexibility in the order of presentation, and partly because of the ways the material can be integrated. The linear algebra, for example, can be merged with the calculus to form a four-semester sequence, or the differential equations can be merged with calculus. Thus, if an institution cannot include all this work in the first two years, it can postpone to the upperclass years part of the development of the calculus of several variables, or the linear algebra, or the differential equations.

However, in any arrangement a modicum of the algebra of vector spaces should be incorporated into a calculus course both to gain a richer formulation of the basic analysis and to provide an economical, simplifying notation. Similarly, we recommend the introduction of appropriate geometric and topological concepts (such as intervals, open and closed sets, neighborhoods, compactness) to illustrate the power of the mathematics under study, to add to its understanding, and to develop the student's intuition.

II. Higher Undergraduate Mathematics

This part of the program will be discussed from two viewpoints: that of the college or university department of mathematics, and that of the student. Every college department offering a major in mathematics should endeavor to provide the following courses at the upper division level:

Complex analysis
Abstract algebra
Geometry-topology
Probability or mathematical physics
Real analysis

Not every student will take all of these courses; choices will depend on the student's intent.

In addition, to achieve a richer and more comprehensive program, a department should offer, as far as its resources will permit, a balanced selection of courses with topics selected from:

Algebra
Analysis
Applied mathematics (in both the natural and social sciences)
Foundations and logic
Geometry (algebraic, differential, or projective)
Mathematical statistics
Number theory
Topology

Secondly, from the viewpoint of the student, the following principles should be observed:

(a) that in his upperclass years, at least three of the following four categories should be represented in his course program: (1) algebra; (2) analysis; (3) applied mathematics; (4) geometry-topology.

(b) that included in his program there should be, in order to achieve depth, at least two full year courses—that is, courses in which the first semester is an essential prerequisite to the second.

(c) that a student majoring in mathematics should have at least seven semester courses beyond our suggested introductory college mathematics.

INITIATIVE AND INDEPENDENCE

A description of the mathematical content of a sound contemporary undergraduate program in mathematics is only part of the story: a large part, to be sure, but not the whole story. For mathematics is not just so many definitions, postulates, and theorems to be learned by the student in a passive fashion. There is also the active side, with a student gradually becoming independent of his professors until he finally becomes a colleague. Thus, the student must increasingly take the initiative, not only to construct proofs by himself, but also to develop his imaginative powers so that he can make conjectures for proof or disproof, even perhaps going on to contribute more fully by creating new concepts and theories that he adds to the world of mathematics.

This process has its beginning in a very small way when the student solves textbook problems. Another component is added when the student learns to read the textual material by himself, later making the passage to the reading of papers in the journals, which are more compactly written and therefore more difficult to read. There are other ways in which the undergraduate student can develop his initiative. Some schools pay a lot of attention to various ways of doing this, others very little. We urge that every school face this question, developing some appropriate scheme. Without attempting an exhaustive list, we mention a few common patterns.

There are seminars and colloquia of one kind or another wherein the student makes reports. There is the undergraduate thesis in which a student makes a contribution, original for him but not usually original in the larger sense. There is the developmental course, a version of the Socratic method, in which the student is led to develop a body of mathematical material under the skillful guidance of the professor. The number of teachers who employ the developmental method in its full-blown form is not large, but they are an enthusiastic and dedicated band of people in their devotion to this procedure. A partial, or modified, use of the developmental method is employed widely. It would be desirable for all schools to give attention to the problem of enrolling the student actively in the study of mathematics beyond a mere passive attendance at lectures.

PRESENT ACTION AND FUTURE PLANS

Free copies of this document are available now and may be obtained by writing to the CUPM Central Office. In addition, a more complete report including detailed outlines of some of the suggested courses will soon be available to assist colleges in undertaking to institute honors curricula.

It is hoped that the program herein outlined will be discussed at mathematics departmental meetings, at section meetings of the Mathematical Association of America, and at meetings of other interested groups.

Conferences will be held in order to explain this program in greater detail and on a more personal basis. Such conferences will be regional in nature, and it is planned that several colleges and universities from each section of the country will be represented.

For the future, CUPM hopes to modify this program in order that the "normal" undergraduate mathematics major might be better prepared in his consideration of a career in mathematics. In the meantime, we hope that the reader will feel free to add his remarks and suggestions. Such correspondence, as well as requests for additional (free) copies of this booklet, for the more complete report, or for outlines of suggested courses, should be addressed to the CUPM Central Office:

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